Optimal Transport Based Satellite State Estimation

Niladri Das*, Raktim Bhattacharya*, Riddhi P. Ghosh# & Bani Mallick#

*Dept. of Aerospace Engineering, #Dept. of Statistics

Abstract

This work provides an alternative approach to address the problem of satellite state estimation and filtering. Optimal Transport based filtering technique is introduced assuming known noise characteristics. We also propose and study a cost function for our Optimal Transport based filtering. This cost function which exploits the circular nature of an orbit improves estimation performance. Among the two phases of filtering: propagation and update, we focus on Bayesian update.

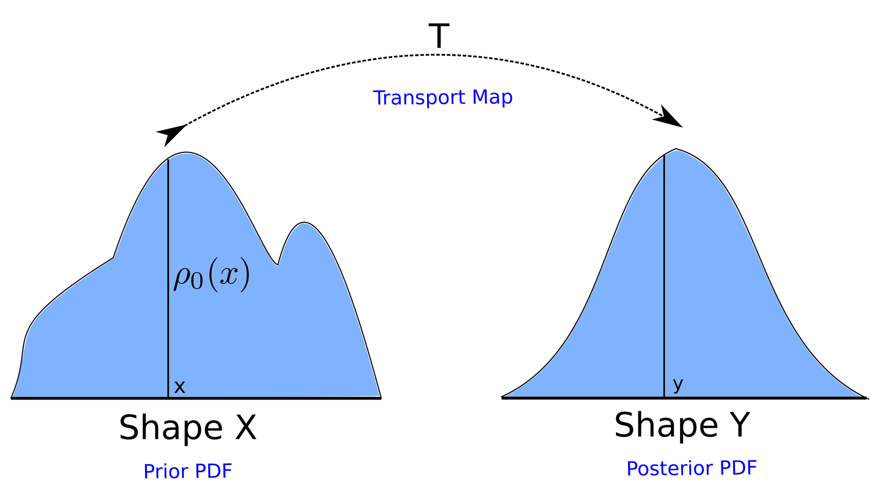
Goals

1. Introduce Optimal Transport Problem
2. Formulate a generic OT based filtering framework and implement it for the 2D satellite problem
3. Study the effects of cost function based on circular nature of the orbit.

Methods

1. Space surveillance is an important issue. It requires proper filtering techniques that addresses the inherent non-linearity of the dynamics and non-gaussianity of the measurement and process noise.
2. Traditional filters such as EKF, UKF and EnKF fails to address this problem. These filters makes local linearity and Gaussianity assumptions
3. Particle Filters aims to alleviate these problems but have its own disadvantages.
4. Optimal Transport Based filtering offers an alternative approach to the same Bayesian filtering problem in a general setting.
5. None of the traditional sensors do not exploit the circularity of the state variable such as in the problem of satellite state filtering.
6 We show how Optimal Transport based filtering coupled with circularity based cost function gives us improved tracking performance.

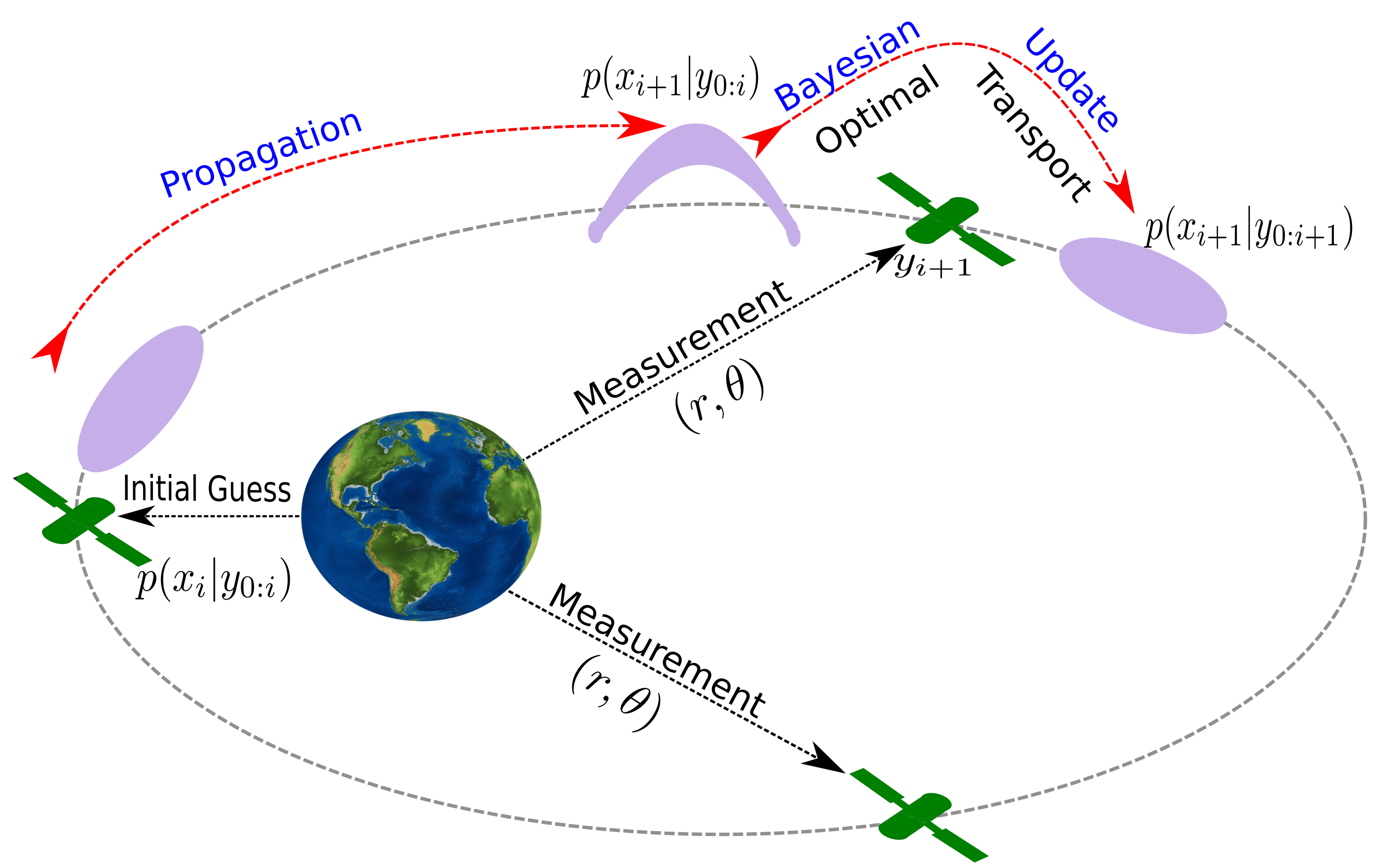
Optimal Transport (OT) Problem



$$T : x \to y$$
$$Cost = \int_{\mathbb{D}} c(x, T(x))\rho_o(x)dx$$

Find T^* so that $Cost$ is Minimum[?]

Evolution of Orbital State pdf



OT Filtering

1. Propagation:

Generate M samples: X_i^g

Propagate these M samples using the dyamics of the system

Forcast: X_i^f

These forcast samples represent the **prior pdf**

2. Update:[?]

$$T^* = arg\min_T \sum_{i=1}^{M}\sum_{j=1}^{M} ||X_i^f - X_j^f||^2 t_{ij}$$
$$P^* = MT^*$$

Assimilation: $X^a = X^f * P^*$

These assimilated samples follow the **posterior pdf**

Conditions on t_{ij}:

$$t_{ij} \geq 0$$
$$\sum_{i=1}^{M} t_{ij} = 1/M$$
$$\sum_{j=1}^{M} t_{ij} = w_i$$

OT Filtering in 2D Satellite Problem

Process Dynamics:

$$M_S(-\ddot{r} + \dot{\theta}^2 r) = \frac{GM_E M_S}{r^2}$$
$$M_S(2\dot{\theta}\dot{r} + \ddot{\theta}r) = 0$$

Measurement Model:

$$y_k = Hx_k + Noise \tag{1}$$

States:

$$r, \dot{r}, \theta, \dot{\theta}$$

Assumptions:

- No process noise, only measurement noise.
- Initial samples are drawn from a Gauss-von-Mises Distribution
- Measurement noise follows a Gauss-von-Mises Distribution

Cost Function:

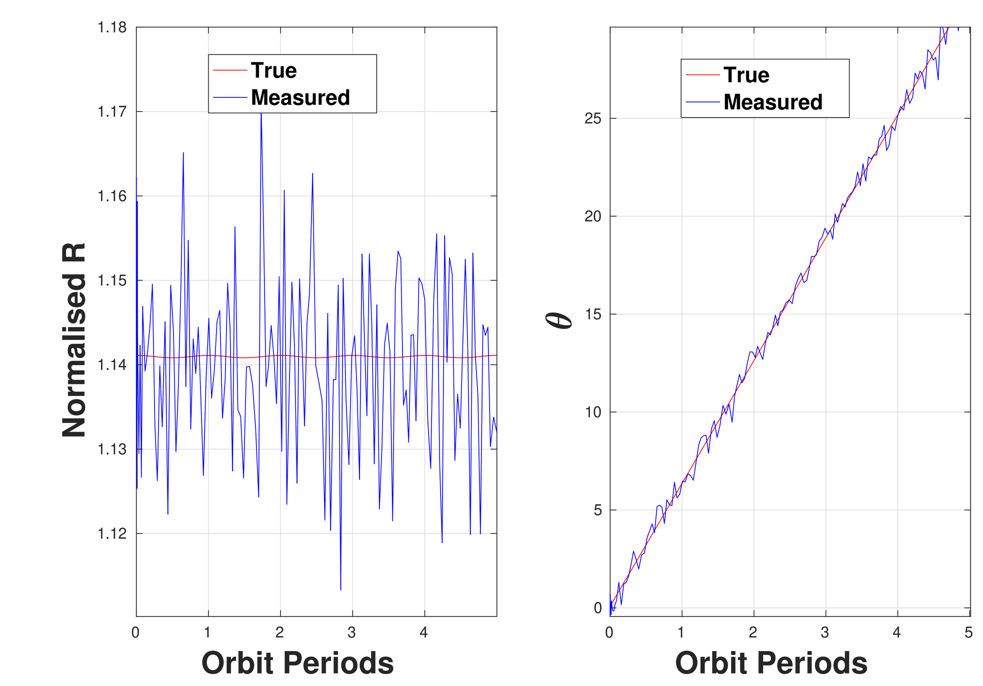
$$norm\begin{bmatrix} r_i - r_j \\ \dot{r}_i - \dot{r}_j \\ \theta_i - \theta_j \\ \dot{\theta}_i - \dot{\theta}_j \end{bmatrix}^2 \to norm\begin{bmatrix} r_i - r_j \\ \dot{r}_i - \dot{r}_j \\ {}^{New}\theta \\ \dot{\theta}_i - \dot{\theta}_j \end{bmatrix}^2$$

where,

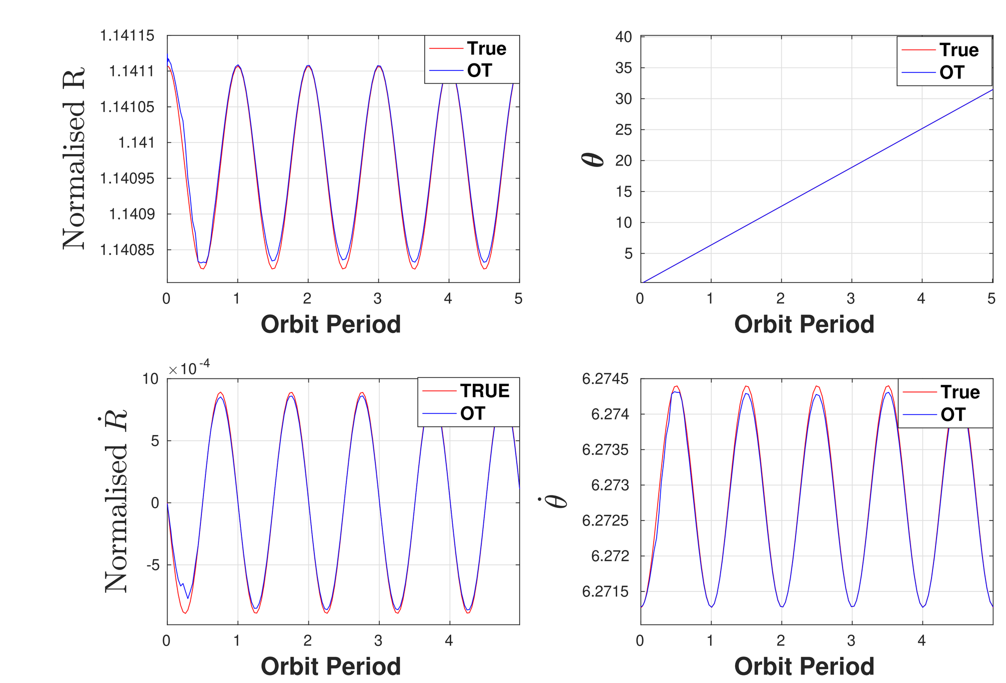
$${}^{New}\theta = min(\theta_i - \theta_j, 2\pi - (\theta_i - \theta_j))$$

Results

Noisy Measurements



OT Filtered State Estimates



Discussions

- **A new method of Bayesian Filtering is presented here.**
- **This Filtering technique is capable of tackling non-linear non-gaussian systems.**
- **For satellite problem using modified distance metric on the circular variable θ improves the state estimate in terms of RMSE.**

References

[1] Nawinda Chustagulprom, Sebastian Reich, and Maria Reinhardt. A hybrid ensemble transform particle filter for nonlinear and spatially extended dynamical systems. *SIAM/ASA Journal on Uncertainty Quantification*, 4(1):592–608, 2016.

[2] Cdric Villani. *Optimal transport : old and new*. Grundlehren der mathematischen Wissenschaften. Springer, Berlin, 2009.

Poster Presented at Texas Systems Day, March 31, 2017

Contact information: Niladri Das, RDMC-201, Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843 – Email: *niladridas@tamu.edu*;

Acknowledgments: This research was supported by AFOSR DDDAS grant FA9550-15-1-0071.